

A Stochastic Methodology for PV System Allocation in Power Distribution Networks.

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Abstract. The increasing penetration rate of photovoltaic generation (PV) in distribution networks has led system operators to face several challenges such as overvoltage, voltage fluctuations, frequency fluctuations, and reverse power flow. This paper introduces a stochastic approach to determine the impact of Hosting Capacity (HC) on a real distribution network in terms of voltage profiles and power losses. The methodology uses historical data of temperature and irradiance over one year, which served as inputs for the stochastic allocation of PV units. Later, PV installation points and the generation capacities are randomly assigned based on historical data. HC is evaluated across various PV penetration rate, analyzing its effects on voltage problems (undervoltage and overvoltage) and power losses. The results demonstrate the effectiveness of PV placement in mitigating voltage issues and identifying the proper HC.

Key words. Hosting Capacity, Monte Carlo simulation, stochastic method, photovoltaic systems, Power Distribution Systems, Power Losses, Voltage Profile, Charging.

Nomenclature

i	: Index of a load.
k	: Index of a branch.
t	: Index of a hour.
$Pen_{PV\%}$	: PV penetration rate.
Pen_{PV}	: Peak power of PV for a penetration rate.
P_{PV_i}	: Peak power of PV Installed in i .
P_{max}	: The maximum total active power of the system over the 24-hour period.
N	: Total number of loads in the network.
$P_{load_{i,t}}$	: Active power of i at time t .
P_{loss_t}	: Total active power losses in the network at time t .
P_{LPVmax}	: Maximum Active power losses in the network during the PV work time.
M	: Total number of branches in the network.
R_k	: Resistance of k .
$I_{k,t}$	: Current flowing through k at time t .
P_{PV_t}	: Total installed active power of all DG units in the network at time t .
N_{PV}	: Total number of distributed generation (DG) units installed.

1. Introduction

The rapid growth of renewable energy sources (RES) has significantly transformed the traditional power grid. Among the RES, photovoltaic (PV) systems have emerged as a key component in the global transition toward sustainable energy solutions [1]. Integrating PV systems into distribution networks offers numerous benefits, including reduced greenhouse gas emissions, enhanced energy efficiency, and decreased dependence on centralized power plants. However, high PV penetration levels introduce substantial operational challenges, such as voltage fluctuations, overvoltage, reverse power flow, and increased power losses, which can compromise the stability and reliability of the grid [2]-[4].

One of the most critical aspects of PV integration is assessing the Hosting Capacity (HC) of distribution networks, which defines the maximum level of PV penetration a network can accommodate without violating operational limits [5]. HC analysis is crucial for grid planning and operation, as excessive PV penetration can lead to voltage instability, power quality issues, and transformer overloading [6]. To address these concerns, researchers have developed various methodologies to estimate HC, broadly categorized into deterministic and probabilistic approaches [7]. Deterministic methods rely on fixed input parameters and worst-case scenarios, often yielding conservative HC estimates. In contrast, probabilistic models incorporate stochastic variations in PV generation, load demand, and network conditions, providing a more realistic and flexible assessment of hosting capacity [8].

To enhance HC estimation and mitigate PV integration challenges, various techniques have been proposed. Optimization-based frameworks integrate voltage regulation mechanisms, demand response strategies, and adaptive control systems to maximize HC while maintaining operational constraints [9].

The paper introduces a stochastic method based on Monte Carlo simulation to estimate the HC in a real Brazilian distribution system. Historical meteorological data (temperature and irradiance) are used to evaluate more realistic PV power-generated profiles. The location and the peak power for each PV system follow a distribution probability based on historical data of PV systems already installed in similar feeders. The proposed methodology enables a detailed analysis of voltage profiles, and power losses under different PV penetration rates. Moreover, the proposed analysis aims to evaluate the effectiveness of PV in mitigating undervoltage issues and reducing power losses, and to identify the potential overvoltage risks under different penetration rates.

2. Data-Driven Hosting Capacity Assessment

Methodology

To accurately analyze the Hosting Capacity (HC) of the network, a thorough assessment of key influencing parameters is essential. An extensive data search was conducted to collect critical information, including temperature, irradiance, and PV system dimensions, ensuring realistic and statistically relevant results. By grouping irradiance and temperature profiles of the network region, a more comprehensive dataset was obtained, improving the reliability of PV system simulations. Additionally, the sizing of PV units was determined through a historical analysis of installations at the medium voltage level, allowing the model to incorporate real-world deployment trends. The data extraction and processing are structured into the following stages.

A. Data Collection.

Historical temperature and irradiance data in a year were analyzed, resulting in average hour profiles for each day of the year, which capture seasonal variations affecting PV generation. Meteorological patterns were examined to enhance regional accuracy and refine the stochastic model. Additionally, the Agência Nacional de Energia Elétrica (ANEEL), provides guidelines for the construction of the Base de Dados Geográfica da Distribuição (BDGD), a georeferenced database representing the real electrical distribution system [10]. For this study, BDGD data were extracted and converted into OpenDSS-compatible models using Python, ensuring accurate representation of the real distribution network for simulation and analysis.

B. PV generation profile.

Monthly average irradiance and temperature profiles were computed to create realistic PV generation profiles. A stochastic selection method was applied to introduce randomness in PV generation profiles, better reflecting real-world variability. These selected profiles were then used to generate load shapes in OpenDSS, allowing for a more precise simulation of daily PV energy production and its effects on the network.

C. PV Size Determination.

The installed PV system sizes at medium voltage level from the last five years were analyzed to establish probabilistic distributions for new installations, ensuring that the sizing of new PV systems aligns with historical trends. Higher probabilities were assigned to PV sizes that have been more commonly deployed, resulting in a realistic distribution of installed capacities. This probabilistic approach enhances the model's accuracy by ensuring that PV size allocations reflect actual installation patterns and expected deployment scenarios.

The proposed methodology creates a robust data-driven framework for PV hosting capacity analysis. This approach enhances HC estimation accuracy by combining real-world network conditions with statistical and probabilistic modeling. Additionally, this methodology enables the evaluation of various PV penetration scenarios, ensuring that network reliability and operational limits are maintained while maximizing renewable energy integration.

3. Stochastic PV Allocation Model

The probabilistic model for PV allocation follows a stochastic process, ensuring a realistic and unbiased distribution of PV systems within the network. This approach integrates historical meteorological data, real network topology and the statistical PV installation trends to create a comprehensive and data-driven methodology. The process ensures that PV generation is allocated dynamically while reflecting real-world variations in irradiance, temperature, and deployment patterns. This method ensures a realistic and unbiased distribution of PV systems.

Fig. 1 shows the first step in the stochastic PV allocation model involves processing historical temperature and irradiance data. Additionally, the historical PV installation data are analyzed to define sizes and deployment trends. The real network topology is then imported into OpenDSS, ensuring that the electrical characteristics, load demands, and network constraints are accurately represented. This enables a baseline simulation that establishes the network's initial voltage profile, power losses, and load distribution before PV integration. The maximum network load over a 24-hour period is determined by:

$$P_{max} = \max \left(\sum_{i=1}^N P_{load_{i,t}} \right), t \in [0,24] \quad (1)$$

Where i is index a load $\in N$ and t is index of hour. P_{max} is power value used as a reference for the definition of PV penetration rate. $P_{load_{i,t}}$ is the load in i at time t .

The total PV capacity that will be integrated into the network for a penetration rate Pen_{PV} is determined in (2) based on the selected penetration rate, $Pen_{PV\%}$, of P_{max} :

$$Pen_{PV} = P_{max} * Pen_{PV\%} \quad (2)$$

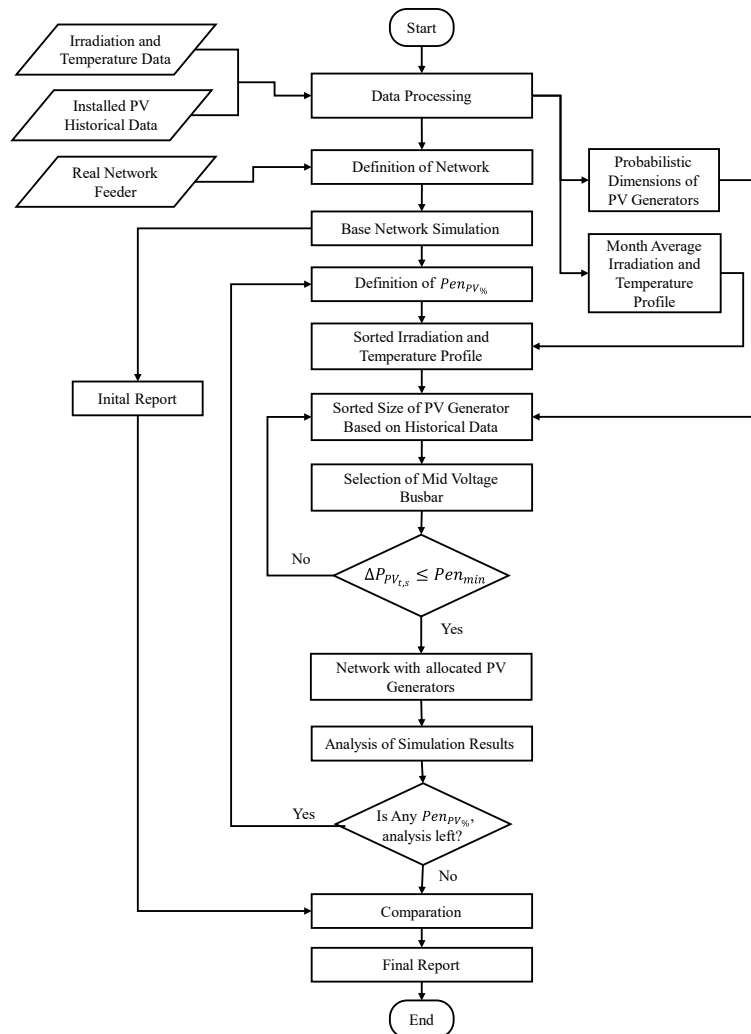


Fig. 1. Flowchart of Stochastic PV allocation Model.

The stochastic PV allocation process follows an iterative structure for different values of $Pen_{PV\%}$. The irradiance and temperature profile are randomly chosen, ensuring variability in PV generation conditions. This methodology follows these steps:

Step 1: Select a $Pen_{PV\%}$

Step 2: Random PV Size Determination: The historical probability distribution of installed PV capacities is used to determine the size of the new PV system.

Step 3: Random Busbar Selection: A random mid-voltage busbar is chosen for PV allocation. This process ensures a stochastic spatial distribution, preventing clustering and enabling a realistic representation of decentralized PV deployment.

Step 4: Cumulative PV Capacity Check: The total allocated PV capacity is iteratively updated:

$$P_{PV_{t,s}} = \sum_{i=1}^{N_{PV(s)}} P_{PV,i} \quad (3)$$

Where P_{PV_t} is the cumulative PV capacity at iteration s , and N_{PV} represents the total number of allocated PV

units at s . If $P_{PV_{t,s}} < Pen_{PV}$ go to step 2. Otherwise, to stop the procedure.

Once PV systems are allocated, the modified network is simulated in OpenDSS, analyzing the impact of PV penetration on voltage stability, power losses, and network performance. The bus voltage levels across the network are examined to identify potential issues related to overvoltage or undervoltage conditions due to PV penetration, according to:

$$V_{min} > 0.95 \quad (4)$$

$$V_{max} < 1.05 \quad (5)$$

The active power losses in the system are calculated using:

$$P_{loss} = \sum_{k=1}^M R_k * I_k^2 \quad (1)$$

Then if another $Pen_{PV\%}$ is set, another iteration is executed following the same steps. The results of the stochastic PV allocation simulation at different penetration rates are compared with the initial grid scenario (without new PV allocations), evaluating the network's ability to host PV generation without exceeding operational limits.

4. Results

The proposed methodology is applied to a real distribution shown in Fig. 2. This network is modeled in OpenDSS and represents a typical urban distribution feeder with medium-voltage at 88 kV and 13.8 kV and low-voltage connections. The total system load consists of a combination of residential, commercial, and industrial users. The maximum active and reactive power demand in the network are approximately 4339 kW and 2247 kvar, respectively Fig. 2. The network consists of 404 busbars, forming a radial structure typical of distribution systems. The protection system consists of one circuit breaker and one recloser for the main feeder, and multiple fuses distributed across lateral branches.

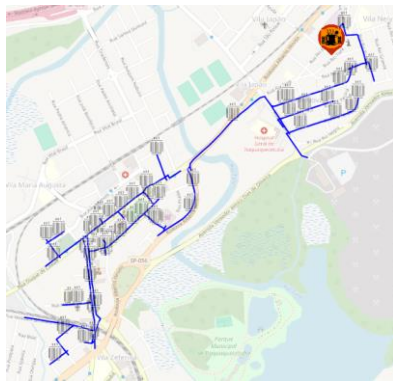


Fig. 2. Real distribution network.

The stochastic PV allocation method is modeled using an interface of Python-OpenDSS using the PVSys component, that integrates both PV modules and inverters into a single representation. The expected PV generation profile is based on historical irradiance and temperature data over one year. This process resulted in 12 representative monthly profiles for irradiance (see Fig. 3) and for temperature (see Fig. 4). These profiles serve as inputs for the stochastic PV allocation model, allowing for an accurate assessment of the network's hosting capacity under varying seasonal conditions.

The study considers various penetration rates, assessing their impact on voltage profile, power losses, and reverse power flow. This real-world feeder model provides a robust test system for evaluating the integration of distributed energy resources, offering valuable insights into grid operation under high PV penetration levels.

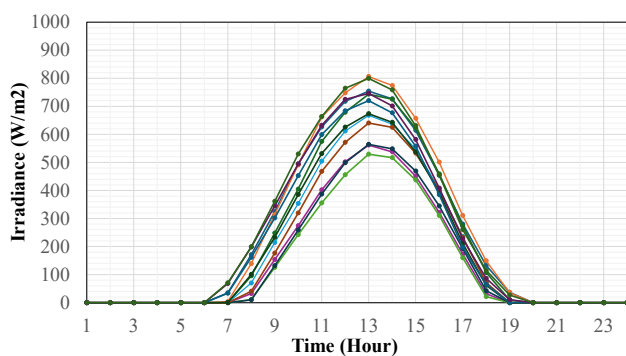


Fig. 3. Average irradiance profile per month.

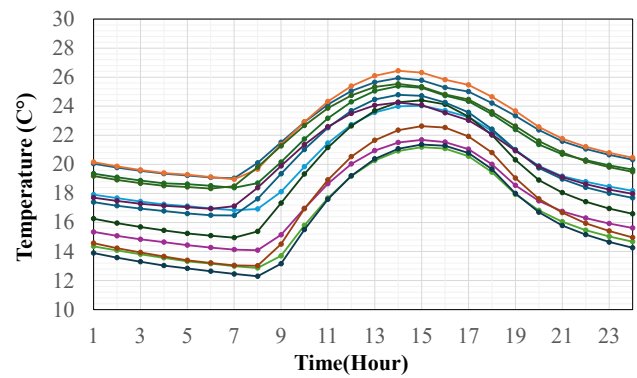


Fig. 4. Temperature Profile per month

A. Analysis without PV.

An initial analysis was conducted before integrating photovoltaic (PV) systems into the network. This assessment includes the active power load profile, the minimum voltage profile, and the active power losses over a 24-hour period. These parameters serve as a reference for comparing the impact of PV integration on the system. P_{max} is obtained analyzing the active power load profile of 24 hrs from Fig. 5. This figure shows a maximum value of 4339 kW at 20h.

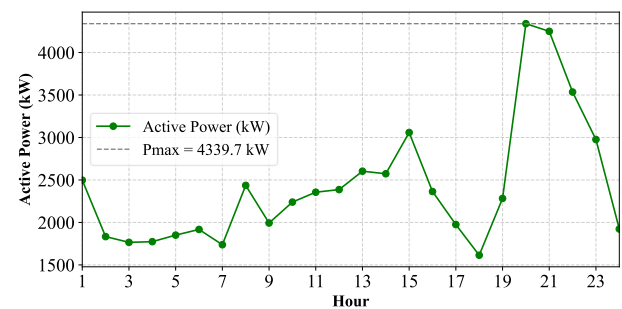


Fig. 5. Power Demand Profile.

Fig. 6 shows the lowest voltage magnitude across all busbars at each hour. At 20h, the grid presents the lowest value of 0.78 p.u.

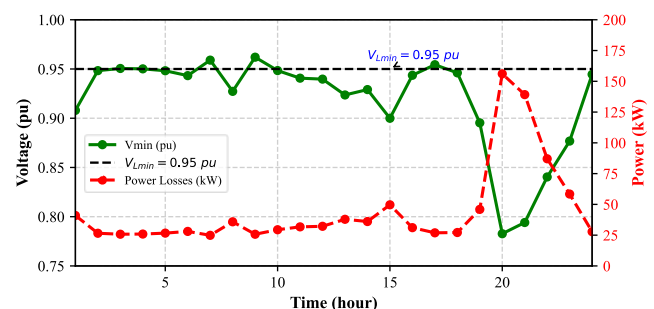


Fig. 6. ITQ feeder power losses and minimum voltage profiles without PVS.

The active power losses were evaluated for the grid at its initial state (without new PV systems). The maximum power losses are obtained at hour 20 with 162.5 kW.

B. Analysis of several PV penetration rates.

A comprehensive analysis was conducted to evaluate the impact of different PV penetration rates on system

operation. This analysis is done only from 8h-17h, available period for PV generation. This assessment focused on voltage profile variations and active power losses. The analysis was designed to provide a realistic assessment of PV hosting capacity by simulating the effects of incremental PV penetration rates from 0% (without new PV systems) to 150% of peak load, with increments of 5. For each penetration rate, fifty scenarios are obtained by Monte Carlo simulation.

Fig. 7 shows a histogram with several PV penetration rates, number of busbars with undervoltage problems, and their respective probability. From figure, the network shows 12 busbars with undervoltage problems during 8h-17h without presence of PV. By increasing the penetration rate, the number of busbars with undervoltage conditions decreases, indicating that the increment of PV systems can improve voltage profiles in certain areas.

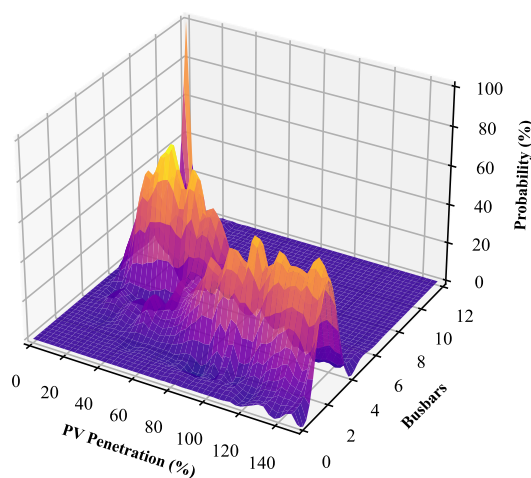


Fig. 7. Histogram of busbars with undervoltage problems between 8h-17h.

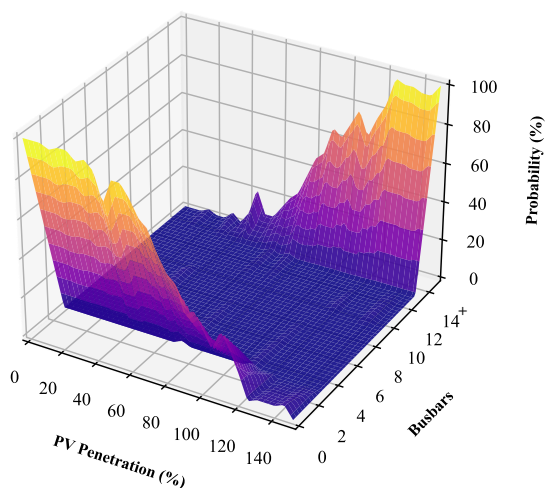


Fig. 8. Histogram of busbars with overvoltage problems between 8h-17h.

Fig. 8 illustrates the probability of busbars experiencing overvoltage as PV penetration rate increases. The initial analysis has no overvoltage problems in any busbar. With low penetration rates, overvoltage risks are minimal. For penetration rate more than 60%, the probability of voltage problem increases. Between 70% and 120%, overvoltage

issues intensify, affecting more than 14 busbars due to excess power injection. Beyond 120% penetration, more than 14 busbars have a near value of 100% probability of exceeding voltage limits, indicating critical areas requiring mitigation.

The Fig. 9 shows the probability of busbars experiencing both undervoltage and overvoltage issues as PV penetration rate increases. At low penetration rates, undervoltage is predominant, but as penetration surpasses 60%, these issues decrease while overvoltage risks start emerging. Between 80% and 120%, voltage problems are more distributed, with some busbars still facing undervoltage while others experience overvoltage due to excess PV generation. Beyond 120% penetration, overvoltage becomes critical, with some busbars reaching 100% probability of exceeding voltage limits.

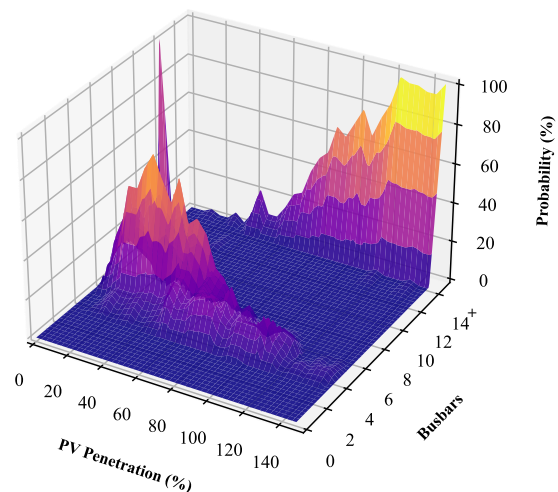


Fig. 9. Histogram of busbars with undervoltage and overvoltage problems between 8h-17h.

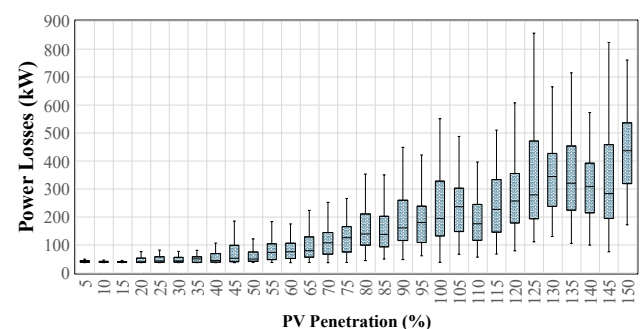


Fig. 10. Boxplot of power losses between 8h-17h for different PV penetration rates.

Fig. 10 illustrates power losses for different PV penetration rates during the period from 8h to 17h. As expected, the increment of PV systems reduces the need for power transmission. Thus, lower power losses are achieved with minimal variation. However, beyond 80% penetration rate, power losses exhibit greater dispersion, indicating that reverse power flow effects significantly the network performance, thus proper control mechanisms have to be included.

These results highlight the dual impact of PV integration on network voltage profile. At low penetration rates, PV

generation effectively mitigates undervoltage issues by supporting local loads. However, as penetration increases, the risk of overvoltage rises, particularly beyond 80%-120%.

This study shows the trade-off between reducing undervoltage and introducing overvoltage risks, emphasizing the importance of strategic PV placement to maximize the hosting capacity. Additionally, the nonlinear relationship between PV penetration rate and power losses reinforces the need for optimized PV allocation and the application of voltage regulation strategies, to prevent excessive network losses at high penetration rates.

5. CONCLUSIONS

This paper introduces a probabilistic approach to estimate the hosting capacity of a real distribution feeder. The findings indicate that moderate PV penetration enhances voltage profiles and reduces active power losses by supplying local demand. However, as PV penetration increases, overvoltage issues emerge, and excessive generation leads to reverse power flow and greater power loss variability. The analysis confirmed that while PV integration reduces losses at controlled PV penetration rates, excessive rates can result in higher losses due to surplus energy injection into the grid. The stochastic allocation model proved effective in capturing real-world uncertainties, highlighting the importance of strategic PV placement and controlled penetration rates to optimize network performance. These results provide valuable insights for grid operators and researchers, offering a data-driven framework for improving PV hosting capacity. Future studies may be focused on optimal allocation strategies, voltage control mechanisms, and energy storage solutions to further enhance Hosting Capacity.

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