and five two-state switches Ka, Kb, Kc, Kd, Ke. The positive pole of the voltage source is denoted as 1 and the negative pole as 0. The switches Ka, Kb, Kc, Kd, Ke are in that order assigned to appropriate phases a, b, c, d, e. Every switch can connect one pole of the voltage source to one assigned phase output.

For that reason, two switching states of every switch are also denoted by 1 and 0. Consequently any state of the inverter model can be described using a set of digits abcde where a, b, c, d, e = {0, 1}. The set of five elements where elements can be denoted by digits 0 or 1 has 32 variations that is: 00000, 00001,…11111. Each inverter state is determined by one five-digit binary number where the order of digits corresponds precisely to the order of phases abcde. By converting binary numbers to the decimal number system it is possible to denote all 32 inverter states by decimals: 0, 110,…3110, respectively. Thus, the state denoted as 16_{10} $(10000)_2$ determines that the phase a is connected to the positive pole of the voltage source, while the phases b, c, d and e—to the negative pole. That is the model example presented in Figure 1. The indices 10 and 2 indicate the base of the number system. This principle which eliminates decimal numbers may be used in other notation systems.

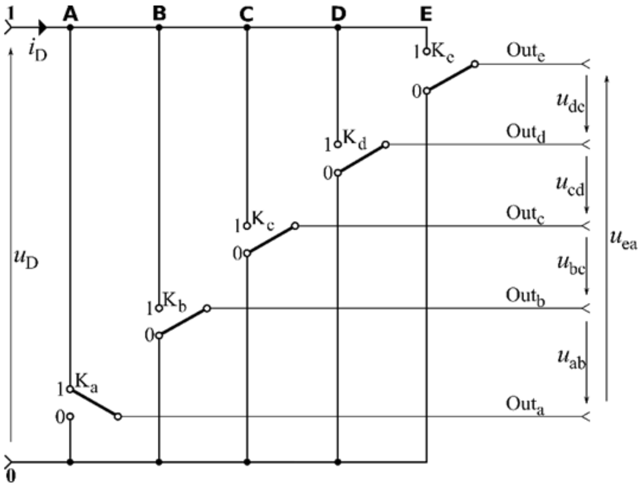

Fig. 1. The model of the five-phase two-level inverter with five switches.

The basic idea of the model depends on the rule of precisely controlled relation "number-phase". This idea may be developed and applied for modelling other multiphase or even multilevel inverters.

B. State-Vector of the Five-Phase Two-Level Inverter

The consecutive states of the five-phase inverter are denoted by the decimal index k = 0, 1, ... 31. Each selected k-state means that five phase-to-phase output voltages are connected to load. These voltages form a matrix row assembled of five elements as a result defining the state vector of the VSI:

$$V_k = [U_{abk}\ U_{bck}\ U_{cdk}\ U_{dek}\ U_{eak}]\ k = 0,1,2,\ldots.31 \quad (1)$$

Since $u_{abk} + u_{bck} + u_{cdk} + u_{dek} + u_{eak} = 0$, then the state vector V_k is determined by any four successive phase-to-phase voltages under the condition that *x* and *y* denote the neighboring phases that is *x*, *y* are: x – a, b, c, d, e and y = a, b, c, d, e.

The binary expansion of the index k permits to determine output polar voltages. They are referenced to the negative pole of the voltage source U_D, for example: $u_{a0k} = a_k u_d$ or $u_{b0k} = b_k u_d$. As a result, the state vector V_k is defined by use of respective binary symbols of the index k. The transpose of the matrix V_k (1), denoted as the state vector V_k, presents all five phase-to-phase voltages:

$$V_k = \begin{bmatrix} u_{abk} \\ u_{bck} \\ u_{cdk} \\ u_{dek} \\ u_{eak} \end{bmatrix} = \begin{bmatrix} u_{a0_k} - u_{b0_k} \\ u_{b0_k} - u_{c0_k} \\ u_{c0_k} - u_{d0_k} \\ u_{d0_k} - u_{e0_k} \\ u_{e0_k} - u_{a0_k} \end{bmatrix} = U_D \begin{bmatrix} (a_k - b_k)_2 \\ (b_k - c_k)_2 \\ (c_k - d_k)_2 \\ (d_k - e_k)_2 \\ (e_k - a_k)_2 \end{bmatrix}. \quad (2)$$

Since the symbols a_k, b_k, c_k, d_k, e_k can assume only values 0 or 1, then the output phase-to-phase voltage, independently of the phase number, may assume only three values: 0, $+U_D$, $-U_D$.

Figure 2 presents a few selected voltage waveforms: three polar and consequential two phase-to-phase voltage waveforms: u_{a0}, u_{b0}, u_{c0}, u_{ab}, u_{ac}.

In order to receive such voltage waves, it was assumed that all phase outputs are connected consecutively, every single 72º, for half a period to the positive pole of the voltage source U_D. The phase-to-phase voltage U_{ac} has been determined according to the rule $U_{x(x+2)k}$. So, the state vector is defining as a matrix composed of one row including five elements (3).

$$V_k = [U_{ack}\ U_{cek}\ U_{ebk}\ U_{bdk}\ U_{dak}] \quad k = 0,1,2,\ldots.31 \quad (3)$$

The relevant rms values are: $U_{ab} \approx 0.63\ U_D$ whereas $U_{ac} \approx 0.89\ U_D$.

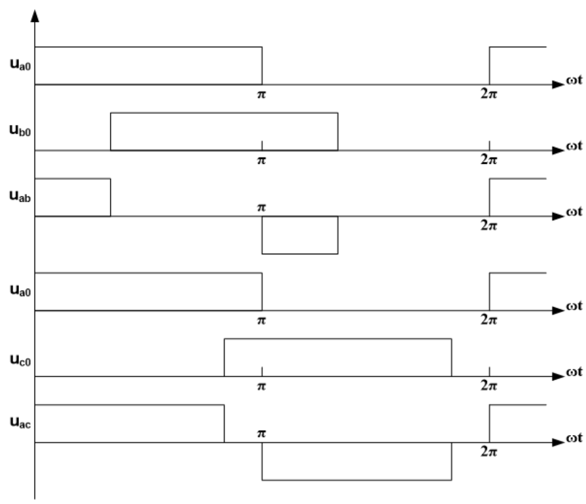

Fig. 2. Polar and phase-to-phase voltage: U_{a0}, U_{b0}, U_{c0}, U_{ab}, U_{ac}.

C. Pentagon and Star Connected Load of the Five-PhaseTwo-Level Inverter

The inverter state vector works on physical quantities and it does not need transformation to be applied for load current calculation. A model of the five-phase two-level inverter with pentagon connected load is presented in

Figure 3. It is assumed that the load is symmetrical and every phase-to-phase load is equal to two-terminal networks: resistor R - inductance L - counter RMF connected in series. Switching the vector on means that five resultant voltages are connected to the load. The situation in Figure 3 articulates that the phase a is connected to the positive pole 1 ($+U_D$), while the remaining phases to the negative pole 0 ($-U_D$). Therefore, the state vector is determined as $V_{16(10000)}$.

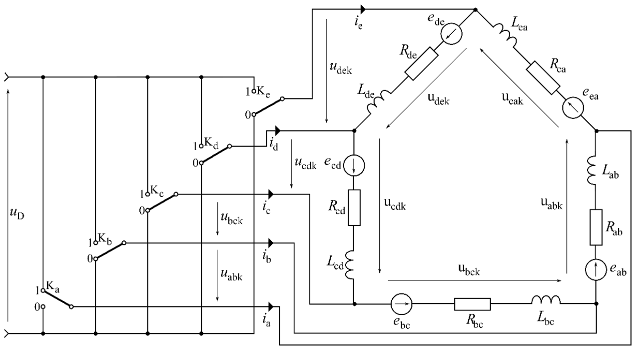

Fig. 3. The model of the five-phase two-level inverter with the pentagon connected load according to the state vector $V_{16(10000)}$.

If the load remains in a pentagon connection, then suitable phase-to-phase voltages are equal to the voltage determined in (2). The pentagon connected load is not considered later in this article.

D. Star Connected Load of the Five-Phase Two-Level Inverter

Figure 4 presents the five-phase two-level inverter model with a load arrangement that is connected in a star. For the considered class of inverters, it can be assumed more than one mathematical definition of the five-phase two-level inverter state vector V_k. For instance, the following definition:

$$V_k = \begin{bmatrix} u_{abk} \\ u_{bck} \\ u_{cdk} \\ u_{dek} \end{bmatrix} == U_D \begin{bmatrix} (a_k - b_k)_2 \\ (b_k - c_k)_2 \\ (c_k - d_k)_2 \\ (d_k - e_k)_2 \end{bmatrix} \quad (4)$$

is effective in view of the fact that $u_{abk} + u_{bck} + u_{cdk} + u_{dek} + u_{eak} = 0$.

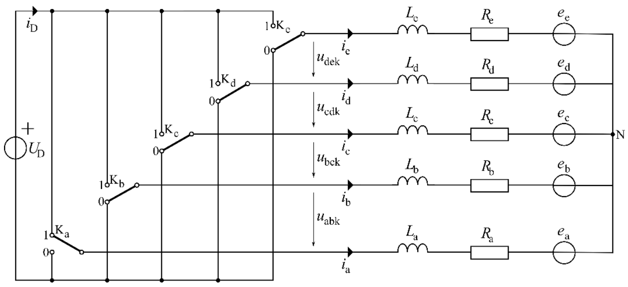

Fig. 4. The model of the five-phase two-level inverter with the load connected in a star.

If at a time point $t=t_n$ the control system switched on the vector V_k at that time, a quadruple of phase-to-phase voltages was connected to the circuit. The selected vector V_k acts in a time interval $<t_n,t_{n+1}>$ and it is assumed - as it was aforesaid - that this interval is very short in relation to a time constant of the circuit and a period of the interphase voltage. Then, the following assumptions might be acceptable:

$$u_{abk} = U_{abk}, u_{bck} = U_{bck}, u_{cdk} = U_{cdk}, u_{dek} = U_{dek}$$
$$e_a = E_a, e_b = E_b, e_c = E_c, e_d = E_d, e_e = E_e \quad (5)$$

It means that in the time interval $t<t_n,t_{n+1}>$ all above voltages are constant. Additional assumptions result in the fact that the system considered in Figure 4 is not equipped with a neutral conductor, so the following dependencies are obligatory (6).

$$\begin{aligned} e_a + e_b + e_c + e_d + e_e &= 0 \\ i_a + i_b + i_c + i_d + i_e &= 0 \\ I_{0a} + I_{0b} + I_{0c} + I_{0d} + I_{0e} &= 0 \end{aligned} \quad (6)$$

If the phase load is fully symmetrical it is profitable to assume the equal opportunity of resistances and inductances as well: $R=R_a=R_b=R_c=R_d=R_e$ and $L=L_a=L_b=L_c=L_d=L_e$. As a matter of fact, the mathematical model of the circuit is described by the system of four equations which are in [13]. Solving the system, it is possible to acquire all analytical phase voltage and current waveforms.

In order to obtain a final solution, it is necessary to introduce expressions describing the relations between phase and phase-to-phase voltages. The applicable interdependence between phase voltage U_x and phase-to-phase voltage U_{xy} does not depend on selected vectors and remains the following:

$$\begin{aligned} 5U_a &= 4U_{ab} + 3U_{bc} + 2U_{cd} + U_{de} \\ 5U_b &= -U_{ab} + 3U_{bc} + 2U_{cd} + U_{de} \\ 5U_c &= -U_{ab} - 2U_{bc} + 2U_{cd} + U_{de} \\ 5U_d &= -U_{ab} - 2U_{bc} - 3U_{cd} + U_{de} \\ 5U_e &= -U_{ab} - 2U_{bc} - 3U_{cd} - 4U_{de} \end{aligned} \quad (7)$$

For instance, the dependence defining the phase voltage U_a can be proved by the use of the appropriate voltage vectors. This situation is illustrated in Figure 6.

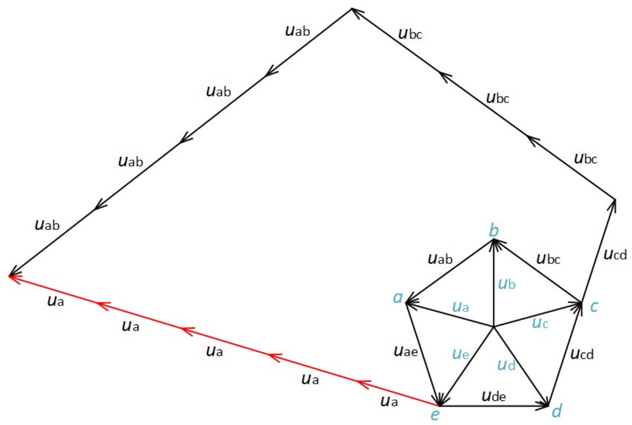

Fig. 6. Mutual dependence between the phase voltage U_a and the appropriate voltage vectors: $5U_a=4U_{ab}+3U_{bc}+2U_{cd}+U_{de}$.

Knowledge of phase voltages allows the solving of the system of equations and finding the successive phase currents. The output phase-to-phase voltage of the two-level inverter, independently of the phase number, may assume only three values: $0, \pm U_D$. Solving the four-loop circuit in the way presented in [12] and converting (7) it is possible to write the generalized expression describing the phase voltages: $u_{ak}, u_{bk}, u_{ck}, u_{dk}, u_{ek}$, of the two-level five-phase VSI as well as phase load currents i_a, i_b, i_c, i_d, i_e. They all can be described using binary digits of the vector index $k = (a_k b_k c_k d_k e_k)_2$ binary expansion:

$$\begin{bmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \\ i_d(t) \\ i_e(t) \end{bmatrix} = \left\{ \frac{U_D}{5} \begin{bmatrix} 4a_k - b_k - c_k - d_k - e_k \\ 4b_k - a_k - c_k - d_k - e_k \\ 4c_k - a_k - b_k - d_k - e_k \\ 4d_k - a_k - b_k - c_k - e_k \\ 4e_k - a_k - b_k - c_k - d_k \end{bmatrix} - \begin{bmatrix} E_a \\ E_b \\ E_c \\ E_d \\ E_e \end{bmatrix} \right\} \frac{1 - e^{-\frac{t}{\tau}}}{R} + \begin{bmatrix} I_{0a} \\ I_{0b} \\ I_{0c} \\ I_{0d} \\ I_{0e} \end{bmatrix} e^{-\frac{t}{\tau}} \qquad (8)$$

where $\tau = \frac{L}{R}$ is a time constant of the phase load, $E_x = E\sin(\omega t_n + \varphi_x)$ and I_{ox} - phase x current in the time point t_n. These expressions describe phase voltages and currents of the two-level five-phase voltage source inverter. Converting similarly Equations (2) and (7) it makes it possible to write expressions describing phase-to-phase voltages and the resulting currents.

D. Star Connected Load of the Five-Phase Two-Level Inverter

The polar voltage space vector of the two-level five-phase VSI is determined analogically to the way presented in previous sections. The definition was established as:

$$\overline{V}_k = \frac{4}{5}\left(u_{a0k} + q u_{b0k} + q^2 u_{c0k} + q^3 u_{d0k} + q^4 u_{e0k}\right) \qquad (9)$$

where $q = e^{j2\pi/5} = \cos\alpha + j\sin\alpha \qquad \alpha = 72^\circ$

$$\overline{V}_k = \frac{4}{5} U_D \left(a_k + b_k e^{j\alpha} + c_k e^{j2\alpha} + d_k e^{j3\alpha} + e_k e^{j4\alpha}\right) \qquad (10)$$

Applying the Euler's formula and introducing interdependencies among the angles as well as symbols a_k, b_k, c_k, d_k, e_k, the space vector is given as follows:

$$\overline{V}_k = \frac{4}{5} U_D \left[\begin{array}{l} a_k + (b_k + e_k)\cos\alpha - (c_k + d_k)\cos\frac{\alpha}{2} + \\ + j\left((b_k - e_k)\sin\alpha + (c_k - d_k)\sin\frac{\alpha}{2} \right) \end{array} \right] \qquad (11)$$

The Equation (11) expresses the space vector which is defined in symbols of the index k binary expansion. The index k is a decimal number discriminating the state vector $k = (a_k, b_k, c_k, d_k, e_k)_2$, where $a_k, b_k, c_k, d_k, e_k = \{0,1\}$.

After transformation the space vector is a complex number which may be represented by the modulus M_k and the argument φ_k. But in the case of the five-phase inverter the situation is more complicated compared to the three-phase inverters. The expression allowing calculation of the space vector modulus is the following:

$$M_k = \frac{4}{5} U_D \sqrt{a_k^2 + b_k^2 + c_k^2 + d_k^2 + e_k^2 + 2\gamma_1 \cos\alpha - 2\gamma_2 \cos(\alpha/2)} \qquad (12)$$

where coefficients γ_1, γ_2 are

$$\begin{array}{l} \gamma_1 = a_k \cdot b_k + a_k \cdot e_k + b_k \cdot c_k + c_k \cdot d_k + d_k \cdot e_k \\ \gamma_2 = a_k \cdot c_k + a_k \cdot d_k + b_k \cdot d_k + b_k \cdot e_k + c_k \cdot e_k \end{array} \qquad (13)$$

The position of the successive vector is determined by its modulus M_k and the shift angle φ_k. In order to determine correctly the shift angle, it might be necessary to use additionally the *arctg* function (14). It results in periodicity of trigonometrical functions and a different period of sine and tangent functions.

$$\varphi_k = arctg \frac{(b_k - e_k)\sin\alpha + (c_k - d_k)\sin\frac{\alpha}{2}}{a_k + (b_k + e_k)\cos\alpha - (c_k + d_k)\cos\frac{\alpha}{2}} \qquad (14)$$

It was assumed at this point that the denominator of the expression (14) did not equal zero. The circumstances, where digits are equal to 0 or 1, denote that the state vector modulus reaches 0 and its shift angle is indeterminate. The relevant two vectors $V_{0(00000)}$ and $V_{31(11111)}$ are usually called "zero vectors" and they do not influence output current.

According to (12) and after calculations, the modulus M_k assumes three different results:

$$\frac{4}{5} U_D, \quad \frac{4}{5} U_D \cdot \sqrt{2 - 2 \cdot \cos\left(\frac{\alpha}{2}\right)}, \quad \frac{4}{5} U_D \cdot \sqrt{2 + 2 \cdot \cos(\alpha)} \qquad (15)$$

In such a case among the 32 vectors there are three groups of 10 active vectors each beside two nonactive zero vectors. The parameters: moduli M_k, coefficients γ_1, γ_2 and shift angles φ_k of all 30 active vectors are collected in [13]. The moduli values are approximated.

Figure 7 presents the active polar space vectors of the five-phase inverter.

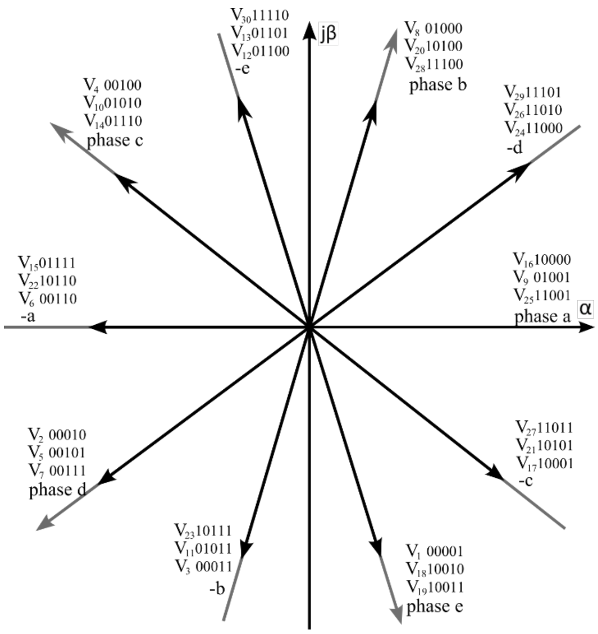

Fig. 7. Mutual dependence between the phase voltage U_a and the appropriate voltage vectors: $5U_a = 4U_{ab} + 3U_{bc} + 2U_{cd} + U_{de}$.

They are presented on the complex plane (α-jβ), where α denotes the real axis and β - the imaginary one. The diagram presents their position. For every successive

$\varphi_k = n\alpha/2\ (n \cdot 36°)$, where n = 0, 1, 2, ... 9 the position of the corresponding three vectors is marked as one vector because they only differ in moduli. The modulus values of each triple are given in order: M_k=0,8 U_D (the upper one), M_k=0,494 U_D, M_k=1,294 U_D, respectively.

3. Experimental Results

This mathematical tool was verified during simulation tests using the PLECS program. Figure 8 presents the main part of the model of the five-phase two-level inverter made in the PLECS program.

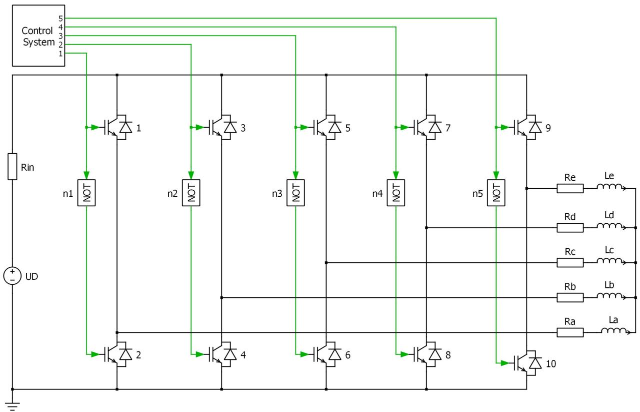

Fig. 8. The model of the five-phase two-level inverter made in the PLECS program.

The following parameters were assumed for the purposes of simulation tests: voltage U_D=600 V, frequency f=50 Hz and load: R=10 Ω and L=15 mH. In the simulation studies, two control strategies were compared with the same parameters of the load circuit, i.e. the strategy described in the article (which was named of vector studies) and PWM (Pulse-Width Modulation).

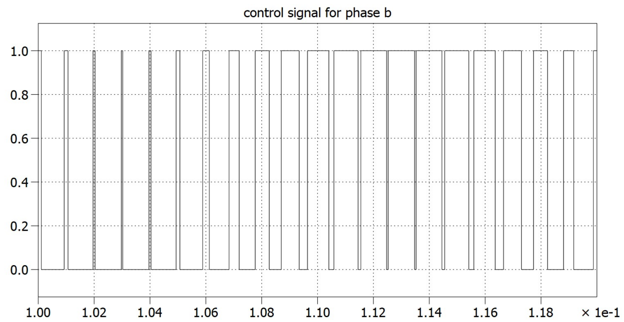

Fig. 9. PWM control signals of phase b.

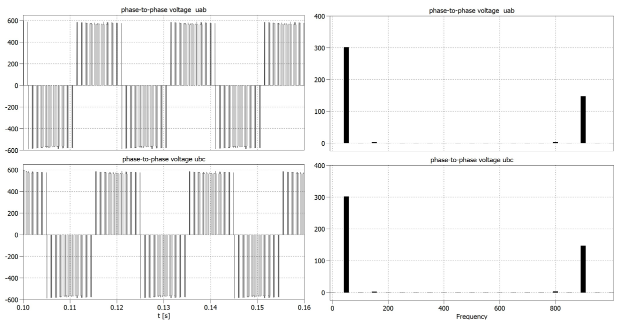

Fig. 10. PWM signals - phase-to-phase voltages of phase ab and bc on the left and their Fourier spectrum on the right, RMS = 257V, THD = 119%.

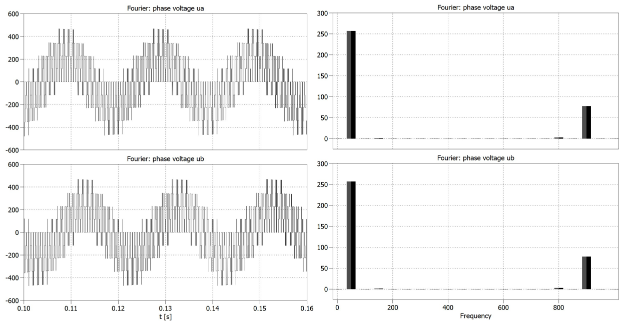

Fig. 11. PWM signals - phase voltages of phase a and b on the left and their Fourier spectrum on the right, RMS = 186 V, THD = 86.2%.

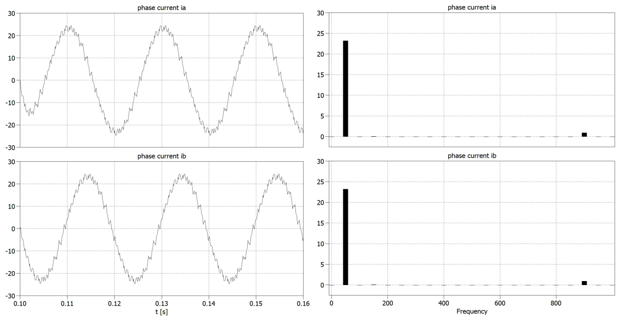

Fig. 12. PWM signals - phase currents of phase a and b on the left and their Fourier spectrum on the right, RMS = 12.3 A, THD = 6%.

Figures from 9 to 12 show the control signals, phase-to-phase voltages as well as phase voltages and currents obtained using PWM control. The RMS and THD were calculated for the inter-phase voltages as well as the phase voltages and currents.

The results (Figures 13 to 16) obtained for the simulations using vector control are presented in a similar order such as for PWM strategy. RMS and THD were also calculated for these data.

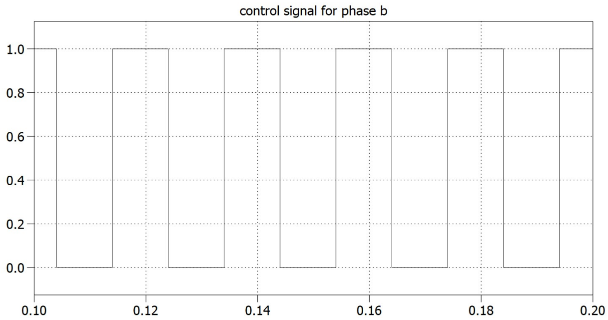

Fig. 13. Vector control signals of the phase b

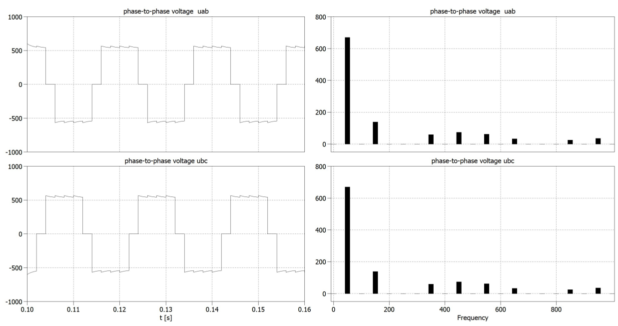

Fig. 14. Phase-to-phase voltages of phase ab and bc on the left and their Fourier spectrum on the right, RMS = 382 V, THD = 30%.

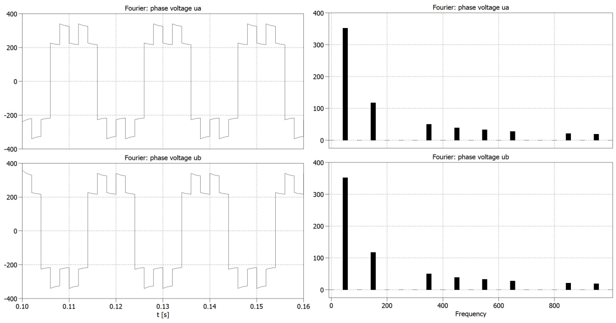

Fig. 15. Vector signals - phase voltages of phase a and b on the left and their Fourier spectrum on the right, RMS = 209.6V, THD = 43%.

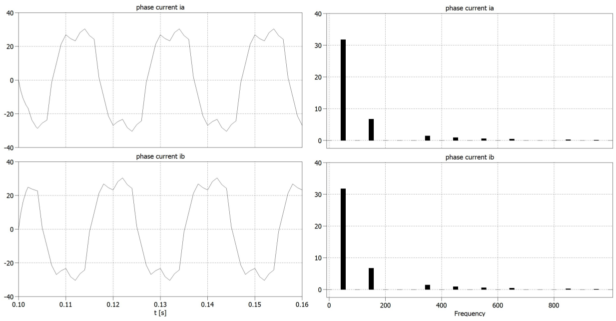

Fig. 16. Vector signals - phase currents of phase a and b on the left and their Fourier spectrum on the right, RMS = 17.4A, THD = 22%.

4. Conclusion

The main contribution of this paper was to present a very simple mathematical system of notation and formulas. The inverter may be defined by the use of definite state vectors or standard space vectors obtained with the use of the polar voltage transform. The paper presents a notation system of all five-phase two-level 32 converter states. Every state is defined by two vectors: state vector and polar voltage space vector. Both vectors are specified by the use of the same digits resulting in binary expansion of the decimal vector index. The method could be advanced to other multiphase and multilevel inverters. This construction of the notation system provides an easy-to-use mathematical tool. It enables selection of a suitable vector sequence assuring the desirable voltage or current waveform. The discussed vector sequences were based on the imperative of only one switching in one phase during the whole voltage period. Three adequate examples were presented in [13]. This mathematical tool makes it easier to define state and space vectors as well as to calculate the available phase and phase-to-phase voltages and the resulting load currents.

This mathematical tool was verified during simulation tests using the PLECS program. The results obtained for steering using PWM and then for steering by use of the described method were compared. By comparing the control signals (Fig. 9 and 13), it can be seen that the described control strategy uses a smaller number of switchings in the same time interval. The inter-phase voltages (Fig. 10 and 14) for the described vector method are characterized by lower RMS and THD values. Similarly, more favorable RMS and THD values can be observed for the phase voltages (Fig. 11 and 15) obtained in the control strategy described in the article. Comparing the phase currents (Fig. 12 and 16), it was noticed that for the PWM control strategy, the current is characterized by a lower THD, but also a lower RMS value.

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