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On the robustness of a multiperiod energy management system including electric vehicles and V2G operation

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Abstract. In recent years, the potential capability of plug-in electric vehicles to offset the intermittency of renewable generation is being analyzed widely. In power systems literature, these electric vehicles are often merged into an aggregator, i.e. an agent responsible of their charging/discharging operation. However, the available power from the aggregator is likewise subject to uncertainty. In this paper, a robust linear programming problem is considered to model the power system operation. In order to clarify the influence of the electric vehicles on the solution, only the available power from the electric vehicles aggregator is taken into account. A multiperiod case study is used to show the behavior of the robust optimization framework in the solution of a multiperiod energy management system. The role of the uncertainty level and different criteria to reduce the cost of uncertainty are also analyzed.

Key words

Energy management, electric vehicles, robust optimization, uncertainty level.

1. Introduction

With the increasing awareness of environmental issues, there is a growing interest in the integration of renewable power sources to address electricity generation. Another advantage of the distributed energy sources is that they are able to deliver clean energy at the electric nodes where it will be consumed. However, the intermittency of these sources results in a relevant drawback in the real time operation of the power systems.

One of the most promising alternatives to fight against this intermittent behavior is the use of electric vehicles as distributed generation sources, often referred to as vehicle-to-grid (V2G). A conceptual framework of the integration of electric vehicles as distributed sources may be found in [1] and [2]. The need of an appropriate coordination mechanism to charge and discharge (V2G) the batteries of these electric vehicles are shown in [3] and [4]. From a power system operator point of view, it is crucial to obtain an accurate assessment so that the electric vehicles can aid to the efficiency and reliability of the system. However, a new uncertainty source, the power availability of the batteries, mainly due to the owner's driving pattern, should be considered in the power system model. In [5], there is a congestion management model, based on the Point Estimate Method [6], which includes electric vehicles and renewable power sources. In that paper, it is assumed that the distribution functions of the input random variables are known.

Robust optimization [7]-[8] is a modeling technique that seeks to minimize the negative impact of future events when the distributions of the input random variable are unknown. An optimal energy management of a small power system with renewable sources and V2G, based on a robust optimization approach, may be found in [9]. That paper includes the wind uncertainty under a multi-scenario approach, while the uncertain power availability by the electric vehicles aggregator is model by adding a robust counterpart to the linear programming problem. A dispatchable generation unit is also considered.

Taking only into account the uncertainty related to the availability of the electric vehicles to charge/discharge power, this paper focuses the behavior of the robust optimization framework in the solution of a multiperiod energy management system. It is assumed that the power system operator controls the charging/discharging operation of a fleet of electric vehicles that are merged together into an aggregator. The role of the uncertainty budget and the different criteria to reduce the cost of uncertainty are also analyzed. From a mathematical point of view, the energy management system is formulated as a robust linear programming problem, where the charging/discharging operation from/to the aggregators is incorporated to the optimization problem through a robust counterpart.

In Section 2, the formulation of the robust linear programming problem is presented. In Section 3 the role of the uncertainty level and the different criteria to reduce the cost of uncertainty are also analyzed. A 24-hour case study to show the results is presented in Section 3. Finally, some conclusions are drawn in section 4.

2. Robust linear programming problem

The model presented in this section is a linear programming robust optimization problem. In order to maximize the clarity of the presentation, the deterministic version of the model is firstly formulated in subsection A, and then, the details of the robust counterpart are presented in subsection B.

The model only takes into account the uncertainty associated with the electric power available to charge or discharge at/from every aggregator in every period (hour). The power system network is incorporated as a linear DC power flow model. It is assumed that the power system operator controls the charge and discharge of the aggregator, but the commitment state of each aggregator (charging, discharging or idle) in every period is previously defined.

A. Deterministic model (non-robust)

The deterministic version of the problem is defined as the linear programming presented below:

$$Min \quad F = \sum_{\forall t} C_{grid}(t) p_{grid}(t) + \sum_{\forall t} \sum_{\forall a} C_{agg}(a,t) p_{agg}(a,t) \tag{2.1}$$

subject to

$$\tilde{p}(n,t) + \sum_{a \in G_n} p_{agg}(a,t) + \sum_{k:r(k)=n} p_{flow}(k,t) - \sum_{k:s(k)=n} p_{flow}(k,t) = \sum_{l \in L_n} L(l,t) \quad \forall n, \forall t \tag{2.2}$$

$$\tilde{p}(n,t) = \begin{cases} p_{grid}(t) & if \text{ n connected to grid} \\ 0 & \text{otherwise} \end{cases} \tag{2.3}$$

$$p_{flow}(k,t) = B_k \left(\delta_{s(k)}(t) - \delta_{r(k)}(t) \right) \quad \forall k, \forall t \tag{2.4}$$

$$\underline{\delta}_n \leq \delta_n(t) \leq \overline{\delta}_n \quad \forall n, \forall t \tag{2.5}$$

$$\underline{P}_{grid}(t) \leq p_{grid}(t) \leq \overline{P}_{grid}(t) \quad \forall t \tag{2.6}$$

$$-\overline{P}_{flow}(k,t) \leq p_{flow}(k,t) \leq \overline{P}_{flow}(k,t) \quad \forall k, \forall t \tag{2.7}$$

$$\underline{P}_{agg}(a,t) \leq p_{agg}(a,t) \leq \overline{P}_{agg}(a,t) \quad \forall a, \forall t \tag{2.8}$$

The objective function to be minimized is the total cost of the energy management system (2.1). In that formulation, upper-case symbols are used to denote constants and parameters, while lowercase ones denote variables. The main variables to be determined are the output/input power from/to aggregators and form/to the main grid. Constraints (2.2) ensure the nodal power balance at every bus at every hour, while (2.3) is the definition of an auxiliary variable to include the power injected from/to the grid at the (only) bus connected to it. Constraints (2.4) include the linearized power flow equations through every line at every hour. Finally, constraints (2.5), (2.6), (2.7) and (2.8) represent the lower and upper bounds to the voltage angles, power from/to the grid, power flow through the power lines, and power from/to the aggregators, respectively.

B. Robust counterpart

The robust counterpart solution is the best uncertainty immunized solution that can be associated with the uncertain problem. The distribution functions of the uncertain data are assumed to be unknown, but their limits are assumed to be known. The goal is to be protected for all allowable realizations of the electric power available for aggregators.

Since the charging/discharging (or idle) commitment state of the aggregators is previously defined, and the expected power from/to the aggregators ($\boldsymbol{P_{AV}(a,t)}$), i.e. the central point of the interval of uncertainty, is also known, two main different behaviors: periods in which they are injecting power to the power system aggregators ($\boldsymbol{P_{AV}(a,t) > 0}$), and those in which they are demanding power from it ($\boldsymbol{P_{AV}(a,t) < 0}$). Hence, constraint (2.8) may be reformulated as in (2.9)-(2.14):

- $if \quad P_{AV}(a,t) > 0$ (Vehicle to grid)

$$\underline{p}_{agg}(a,t) \leq p_{agg}(a,t) \leq P_{AV}(a,t) \quad \forall a, \forall t \tag{2.9}$$

$$\underline{p}_{agg}(a,t) \geq 0 \quad \forall a, \forall t \tag{2.10}$$

$$\underline{p}_{agg}(a,t) = \omega_G P_{AV}(a,t) \quad \forall a, \forall t \tag{2.11}$$

(2.9-2.11) Represent the behavior of the aggregator as a generator.

- $if \quad P_{AV}(a,t) < 0$ (Grid to vehicle)

$$P_{AV}(a,t) \leq p_{agg}(a,t) \leq \overline{p}_{agg}(a,t) \quad \forall a, \forall t \tag{2.12}$$

$$\overline{p}_{agg}(a,t) \leq 0 \quad \forall a, \forall t \tag{2.13}$$

$$\overline{p}_{agg}(a,t) = \omega_D P_{AV}(a,t) \quad \forall a, \forall t \tag{2.14}$$

Equations (2.9)-(2.10) and (2.12)-(2.13) represent the new bounds, depending on the behavior of the aggregator (sign of the variable $p_{agg}(a,t)$), and (2.11) and (2.14) include more restrictive lower and upper bounds to (2.9) and (2.12) respectively.

Parameter $\boldsymbol{P_{AV}(a,t)}$ is modeled as a symmetric and bounded random variable that take values in the interval $[\boldsymbol{P_{AV}(a,t) - \hat{P}_{AV}(a,t), P_{AV}(a,t) + \widehat{P_{AV}}(a,t)}]$, with the parameter $\boldsymbol{\sigma}$ indicating the deviation from expected value, i.e. $\widehat{\boldsymbol{P_{AV}}}(\boldsymbol{a},\boldsymbol{t}) = \boldsymbol{\sigma}\, \boldsymbol{P_{AV}}(\boldsymbol{a},\boldsymbol{t})$.

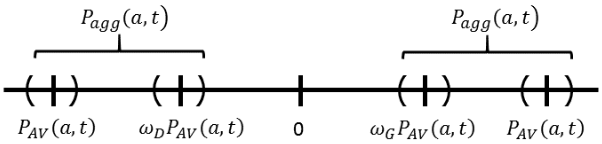

Fig.1. Bounds and robust interval for variable $P_{agg}(a,t)$.

Parameters ω_D and ω_G set the lower bound to generation, when it is necessary for $P_{AV}(a,t) > 0$ and the upper bound to aggregators' demand for $P_{AV}(a,t) < 0$. The use of these parameters it is showed in Fig. 1.

Taking into account the duality properties and linear equivalences [11], the robust counterpart of the deterministic problem in (2.1)-(2.8) is formulated by equations (2.1)-(2.7), and (2.15)-(2-26).

- $if \quad P_{AV}(a,t) > 0$ (Vehicle to grid)

$$-p_{agg}(a,t) + \omega_G P_{AV}(a,t) x(a,t) + \Gamma(a,t)\, z_{G_{LO}}(a,t) + \alpha_{G_{LO}}(a,t) \leq 0 \quad (2.15)$$

$$z_{G_{LO}}(a,t) + \alpha_{G_{LO}}(a,t) \geq \sigma\, \omega_G P_{AV}(a,t) x(a,t) \quad (2.16)$$

$$p_{agg}(a,t) - P_{AV}(a,t) x(a,t) + \Gamma(a,t)\, z_{G_{UP}}(a,t) + \alpha_{G_{UP}}(a,t) \leq 0 \quad (2.17)$$

$$z_{G_{UP}}(a,t) + \alpha_{G_{UP}}(a,t) \geq \sigma\, P_{AV}(a,t) x(a,t) \quad (2.18)$$

- $if \quad P_{AV}(a,t) < 0$ (Grid to vehicle)

$$p_{agg}(a,t) - \omega_D P_{AV}(a,t) x(a,t) + \Gamma(a,t)\, z_{D_{UP}}(a,t) + \alpha_{D_{UP}}(a,t) \leq 0 \quad (2.19)$$

$$z_{D_{UP}}(a,t) + \alpha_{D_{UP}}(a,t) \geq -\sigma\, \omega_D P_{AV}(a,t) x(a,t) \quad (2.20)$$

$$-p_{agg}(a,t) + P_{AV}(a,t) x(a,t) + \Gamma(a,t)\, z_{D_{LO}}(a,t) + \alpha_{D_{LO}}(a,t) \leq 0 \quad (2.21)$$

$$z_{D_{LO}}(a,t) + \alpha_{D_{LO}}(a,t) \geq -\sigma\, P_{AV}(a,t) x(a,t) \quad (2.23)$$

- Additional constraints

$$z_{G_{LO}}(a,t), \alpha_{G_{LO}}(a,t), z_{G_{UP}}(a,t), \alpha_{G_{UP}}(a,t) > 0 \quad (2.24)$$

$$z_{D_{LO}}(a,t), \alpha_{D_{LO}}(a,t), z_{D_{UP}}(a,t), \alpha_{D_{UP}}(a,t) > 0 \quad (2.25)$$

$$x(a,t) = 1 \quad \forall a, \forall t \quad (2.26)$$

Equations (2.15)-(2.16) include the robust formulation of the lower bound of the power generated by the aggregator, while (2.17)-(2.18) present the robust formulation of the upper bound of the power generated by the aggregators. The term $\Gamma(a,t)\, zG(a,t) + \alpha G(a,t)$ provides the necessary protection of the robust constraint by maintaining a gap between $P_{agg}(a,t)$ and $P_{AV}(a,t)$. Variables $z_G(a,t)$ and $\alpha_G(a,t)$ are dual variables for each bound of the power generated by the aggregators. Auxiliary variables $x(a,t)$ are needed to formulate the robust problem. Accordingly, equations (2.18)-(2.19) include the robust formulation of the upper bound of the power demanded by the aggregators, while (2.20)-(2.21) present the robust formulation of the lower bound of the power demanded by the aggregators, and variables $z_D(a,t)$ and $\alpha_D(a,t)$ are dual variables for each bound of the power generated by the aggregators. Finally, (2.24) and (2.25) set the non-negativity of the dual variables.

It should be noted that equations (2.15)-(2.26) are set only for those periods in which the aggregators are either generating or demanding power, while in those periods in which they are in the idle state, variables $p_{agg}(a,t)$ are set to zero.

The parameter $\Gamma(\boldsymbol{a},\boldsymbol{t})$ takes a value in the interval (0,1). The role of this parameter is to adjust the robustness of the proposed method against the level of conservatism of the solution

3. The uncertainty level

It is widely assumed that the robust optimization model presented by Soyster [10] may be too conservative. In 2004, Bertsimas and Sim [11] formulated a robust optimization problem in which an adjustable parameter can be used to handle the trade-off between reliability and economy, i.e. between the robustness of a solution and its associated cost. That parameter is referred to as uncertainty level or uncertainty budget. In addition, some upper bounds on the probability of constraint violation are also provided in [11].

In this paper the role of the uncertainty level (Γ) is analyzed. Three different situations may be considered for a set of homogeneous uncertainty levels:

- If $\Gamma(\boldsymbol{a},\boldsymbol{t}) = \boldsymbol{0}$ for all aggregators and periods, the robust counterpart problem is equivalent to the deterministic one, i.e. none uncertainty is considered. The solution of the optimization would be the cheapest but, since in the robust problem formulated there is only one random variable for each aggregator and period, half of the interval would be unprotected against the possible realizations of the random variable with uncertainty.

- If $\Gamma(\boldsymbol{a},\boldsymbol{t}) = \mathbf{1}$ for all aggregators and periods, it would be completely protected against all possible realizations of the uncertainties, but it would be the most expensive solution. It is the so-called minimum regret formulation.

- Finally, if this parameter is homogenously fixed (i.e. for all aggregators and periods) to an intermediate value ($\mathbf{0} < \Gamma(\boldsymbol{a},\boldsymbol{t}) < \mathbf{1}$), the robustness of the solution would be linearly decreased, but its cost would be also decreased accordingly.

In this paper, the role of a specific set of this parameter for each aggregator and period is analyzed. It may be used to check, for instance, what would be the increase of the objective function if any (or several) period, or aggregator, is protected over (higher $\Gamma(\boldsymbol{a},\boldsymbol{t})$) a mean value. In order to show the behavior of this specific set of the uncertainty level, the linear programming problem in (3.1)-(3.4) is formulated. The main idea behind that problem is that, although some periods are overprotected and some others are under protected, the total amount of protected power is not modified.

$$Min\ P = \sum_{\forall t} \sum_{\forall a} \left[C_{agg}(a,t) - C_{grid}(t)\right] p_{agg}^{\Gamma_{adj}}(a,t) \qquad (3.1)$$

subject to

$$\sum_{\forall t} \sum_{\forall a} \left| p_{agg}^{\Gamma=1}(a,t) - p_{agg}^{\Gamma_{adj}}(a,t) \right| = \sum_{\forall t} \sum_{\forall a} \left| p_{agg}^{\Gamma=1}(a,t) - p_{agg}^{\Gamma}(a,t) \right| \qquad (3.2)$$

$$p_{agg}^{\Gamma_{adj}}(a,t) = p_{agg}^{\Gamma=0}(a,t) + \Gamma_{adj}(a,t)\left(p_{agg}^{\Gamma=1}(a,t) - p_{agg}^{\Gamma=0}(a,t)\right) \qquad (3.3)$$

$$\underline{\Gamma}_{adj}(a,t) \le \Gamma_{adj}(a,t) \le \overline{\Gamma}_{adj}(a,t) \qquad (3.4)$$

The solution (power from/to the aggregator) of three different case studies is considered as input data in (3.1)-(3.4): the deterministic problem (Γ=0), the minimum regret problem (Γ=1) and the one from a intermediate but homogenous set of the uncertainty level (0<Γ<1). Then, an adjusted uncertainty level set (Γadj) is to be determined. The total is minimized cost in the objective function (3.1). Constraint (3.2) enforced the total amount of protected power of the ‘adjusted solution’ equals the total amount of protected power in the intermediate but homogeneous case study. Since the power from/to the aggregator variable may be positive or negative, depending on the aggregator and period, the absolute value is required. Constraint (3.3) includes the definition of the adjusted uncertainty level set, and (3.4) provides the lower and upper bounds to the uncertainty level in every aggregator and period. It should be mentioned that if the adjusted uncertainty level set of the problem (3.1)-(3.4) is used, as input data in the robust counterpart presented in Section II.B, both problems reach the same solution.

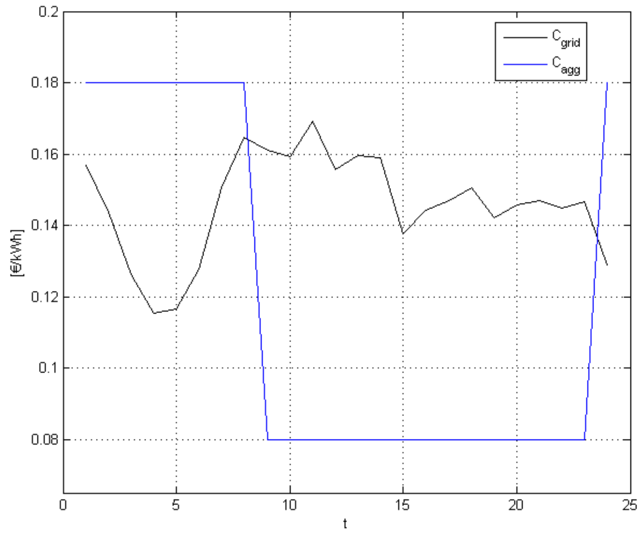

Fig. 2. Energy prices.

4. Case study

A 24-hour multiperiod small-size case study is used to test the proposed strategy. This case study only includes 4 buses, although results presented in this section would be also valid to a higher power system, as long as network constraints do not limit the optimal solution, as in the case studies analyzed. The power demand at every bus and period, independent of the charging/discharging operation of the electric vehicles, and electric energy prices are assumed to be deterministic. The cost of the energy prices for the grid and the aggregators are presented in Fig. 2. These multiperiod cost sets are used for the grid and aggregators, without differencing the direction of the power flow, i.e. from/to the grid/aggregator respectively.

Two aggregators are considered: one of them is located at a bus with a residential load profile (a_1), while the other is at a bus with a commercial load profile (a_2). The uncertainty related to the available power from/to the aggregators is modeled by an expected value and a symmetric robust interval with a constant σ-value of 0.2.

The colored bars in Fig.3 (for aggregator a_1) and Fig. 4 (for aggregator a_2) show the charging/discharging power of each aggregator at each period. Those figures include the results over four different case studies: the deterministic one (blue), the minimum regret solution (red), the intermediate and homogeneous uncertainty level with Γ=0.5 (cyan), and the solution corresponding to an adjusted set of uncertainty level according to the optimization problem described in section 3 (yellow).

Fig. 3 shows the typical behavior of an aggregator (a_1) with a residential load profile: following the electric energy prices, electric vehicles are mainly charging during the night periods and early afternoon, while power is delivered to the system mainly during late afternoon and early evening periods. On other hand, Fig. 3 shows the behavior of an aggregator (a_2) located at a bus with a

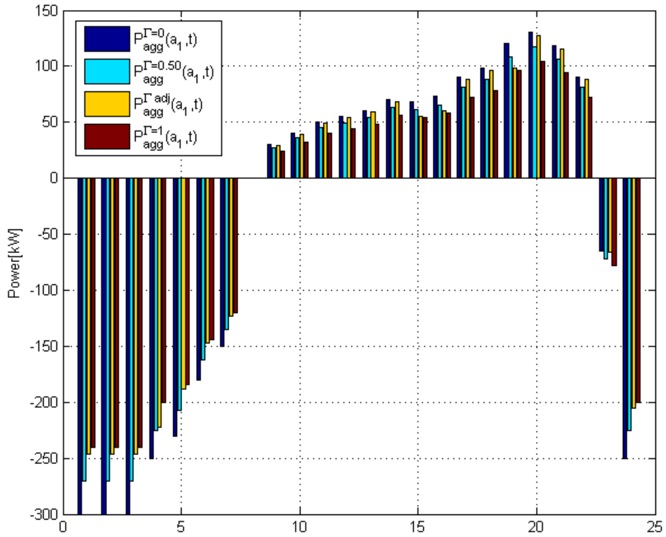

Fig.3. P_{agg} for aggregator 1 for different protection levels.

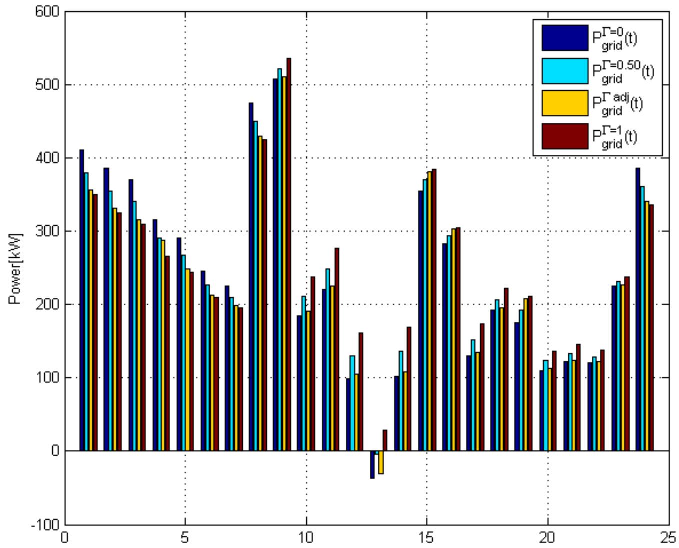

Fig.4. P_{agg} for aggregator 2 for different protection levels.

commercial profile: in an idle state during non-commercial periods and injecting power to the system mainly around noon.

Regarding the uncertainty level, it can be observed, in both figures, that the deterministic solution (Γ=0) and the minimum regret one (Γ=1) reach the extreme values of the injected power from/to the aggregator for all periods; while the homogeneous solution (Γ=0.5) and the adjusted one (Γ_{adj}) remain in intermediate values. The relative position of the extreme value of the deterministic solution and the minimum regret solution depends on the constraint of the robust counterpart that is active: for instance, the power demand of aggregator a_1 (Fig.3) is the highest value, among the four Γ values, in period 24; but it is the lowest value in period 23. The opposite holds for the minimum regret solution in those two periods. It can be also seen that the homogeneous solution (Γ=0.5) is the mean between the deterministic solution and the minimum regret one.

In order to analyze the behavior of the linear programming problem presented in section 3, Fig 3. shows that the power demand of adjusted solution is close to the deterministic solution in period 23, when the price of the power from the grid is higher than the power from the aggregator, while the power demand of adjusted solution is close to the minimum regret solution in period 24, when the price of the power from the grid is lower than the power from the aggregator.

Regarding the power generated by the aggregator, it can be observed in Fig. 3 that the power generated by aggregator a_1 is close to the deterministic solution in most periods from 9 to 22, when the price of the power from the grid is higher than the power from the aggregator, but not in periods 15 and 16, where the solution is close to the minimum regret one. This is due to the fact that the other aggregator, a_2 in Fig. 4, is demanding power in those periods, and hence, the result is the joined effect of both aggregators.

In Fig. 5 the power balance of the power system and the main grid is displayed for each period. The power system is demanding power from the grid in all periods but in period 13. In that period, the power generated by the aggregators is high enough to invert the direction of power and, then, the system is injecting power to the grid; but this is true only in three of the four case studies analyzed: in the solution of the minimum regret problem, the power generated by the aggregators is not high enough and the power system demands power from the grid.

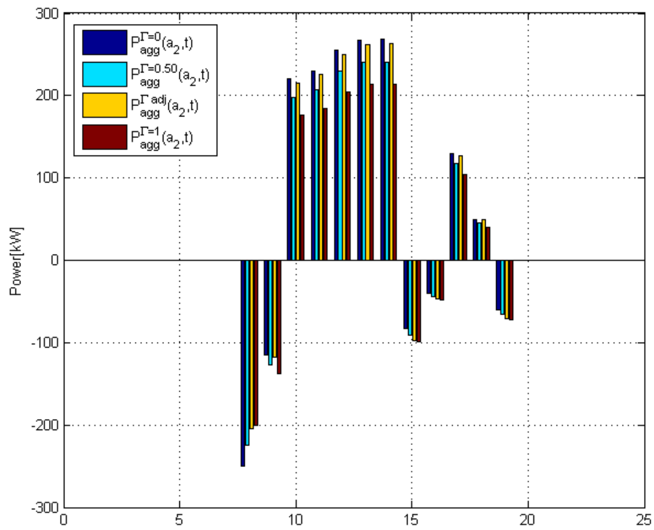

Fig.5. Power from/to main grid for different protection level.

The behavior of constraint (3.2) is shown in Fig. 6: the left-hand side of that equality constraint (red) and its right-hand side (black) are depicted for three different values of the uncertainty level parameter (0.25, 0.5 and 0.75). In may be observed that in some periods the protected power in the adjusted solution (red) is higher than the protected power in the homogenous solution (black), while it is the opposite in other periods, but the total amount of protected power is the same in both solutions.

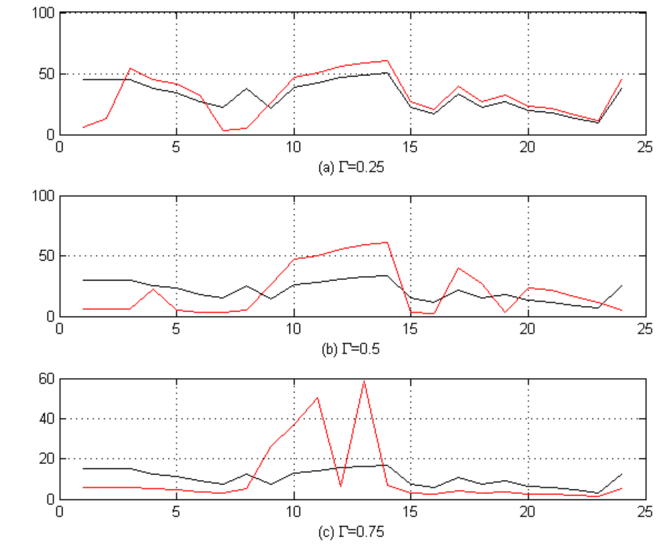

Fig.6. Protection level of the adjusted solution (red) and the homogenous solution (black).

The objective functions of the case studies analysed are shown in Fig. 7. The two extreme points correspond to the deterministic solution (Γ=0) and the minimum regret one (Γ=1), and the linear function joining them (black) represents the evolution of the objective function of problem formulated with an homogeneous uncertainty level. It can be seen that the cost of the adjusted solution (red) is lower than the one from the homogeneous one.

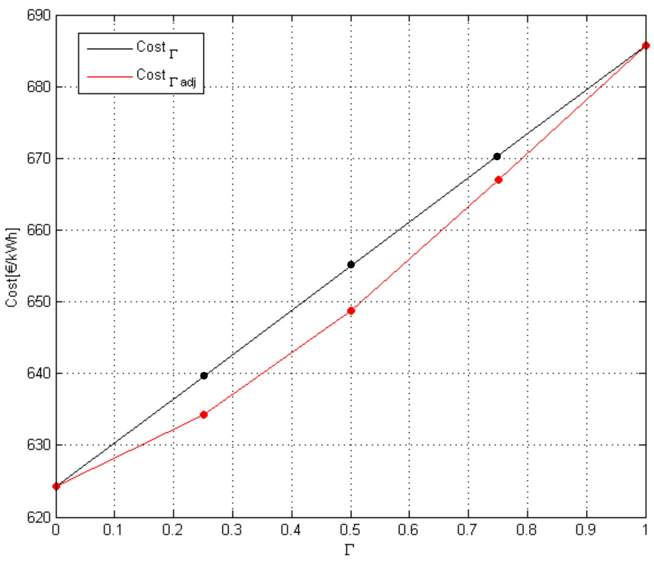

Fig.7. Trend of the optimal object function.

All the optimization problems were solved using GAMS 24.1.3 [13] on a PC with an Intel Core i5 processor and 8 GB of RAM in a few seconds.

5. Conclusions

This paper presents an energy management system formulated as a robust linear programming problem. The only source of uncertainty taken into account is the power availability of the electric vehicles merged into some aggregators. The charging/discharging operation is assumed to be controlled by the power system operator. The most conservative solution and the deterministic one, where the uncertainty is neglected, are compared against some parametrized solutions for different sets of uncertainty levels. A linear programming problem is developed to find out a solution in which the cost of the solution obtained with an adjusted set of uncertainty level, for each aggregator an period, is lower than the cost of the problem formulated with a homogeneous value of the uncertainty level with the same total amount of protected power. Results over a multiperiod case study are presented to show the effectiveness of the proposed procedure.

Acknowledgement

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